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C. WINKELMANN, M. BUSETTO, ABB CORPORATE RESEARCH, THEORETICAL AND COMPUTATIONAL METHODS

A method coupling magneto-statics and eddy current formulations

How to compute equivalent network parameters?

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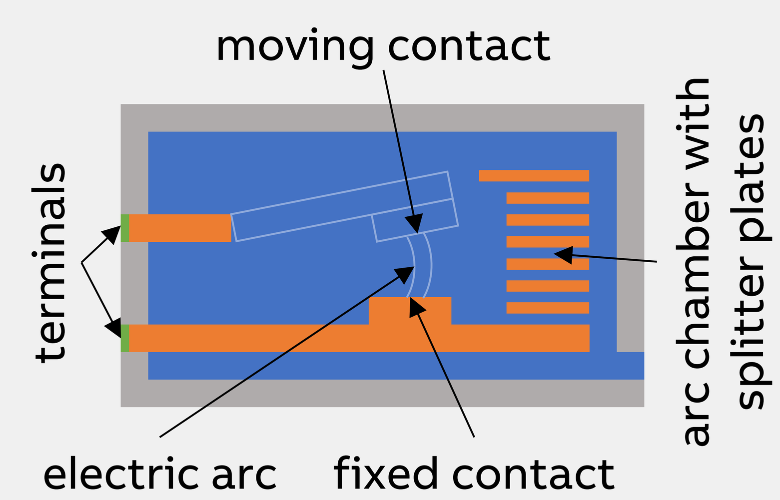
Eddy currents impact performance of circuit breakers but are challenging to simulate. We propose a dedicated formulation.

Eddy Currents in Splitter Plates of Circuit Breakers

- Functioning principle:
  short circuit
  → contacts open
  → plasma arc
- Ohmic losses → arc heats
- Lorentz forces → arc moves
- If arc extinguished → success

- Ferromagnetic splitter plates
  → more Lorentz force on arc
- Eddy currents in splitter plates
  → less Lorentz force on arc



Challenge

- Moving contact → remeshing
- Eddy currents → time derivative
- Both → interpolation?

Approach: Two-domain method

$\Omega = \Omega_E \cup \Omega_M$

- Ω_E eddy currents subdomain
- incl. insulating subdomain Ω_{Ins}
- Ω_M magnetostatic subdomain

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Eddy currents in deforming domain

F O R M U L A T I O N

Strong form, fields

$\underline{\nabla} \times \underline{E} + \partial_t \underline{B} = \underline{0}$ in Ω_E $\quad \underline{\nabla} \times \underline{H} = \underline{j}$

$\underline{\nabla} \times \underline{E} = \underline{0}$ in Ω_M $\quad \underline{\nabla} \cdot \underline{B} = 0$

$\underline{j} = \sigma \underline{E}$ $\quad \underline{B} = \mu \underline{H}$

Strong form, potentials

$\Omega \quad \underline{B} = \underline{\nabla} \times \underline{A}$

$\Omega_E \quad \underline{E} = -\partial_t \underline{A} - \underline{\nabla}\varphi$

$\sigma \partial_t \underline{A} + \sigma \underline{\nabla}\varphi + \underline{\nabla} \times \left(\mu^{-1} \underline{\nabla} \times \underline{A}\right) = \underline{0}$

$\Omega_M \quad \underline{E} = -\underline{\nabla}\varphi$

$\sigma \underline{\nabla}\varphi + \underline{\nabla} \times \left(\mu^{-1} \underline{\nabla} \times \underline{A}\right) = \underline{0}$

Weak form, potentials

$\left(\sigma \underline{\nabla}\varphi, \underline{\nabla}\varphi'\right)_\Omega = 0$ (+ current BC)

$\left(\sigma \partial_t \underline{A}, \underline{A}'\right)_{\Omega_E} + \left(\mu^{-1} \nabla \times \underline{A}, \nabla \times \underline{A}'\right)_\Omega$
$= \left(-\sigma \underline{\nabla}\varphi, \underline{A}'\right)_\Omega$

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Dedicated formulation

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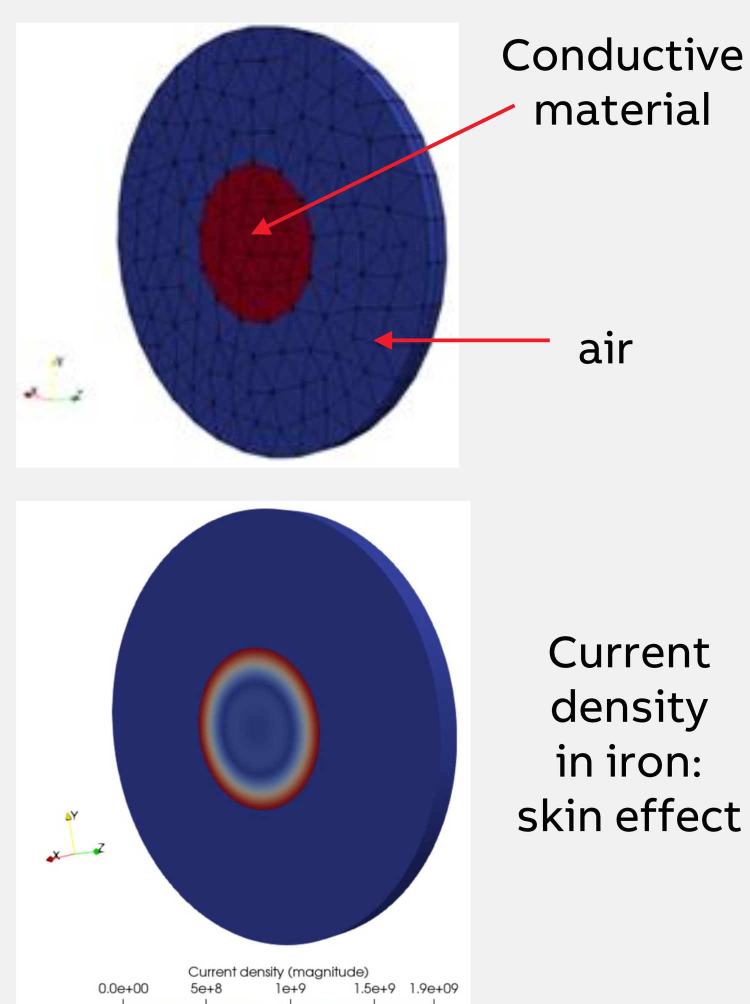
Our formulation does not allow to compute voltage in the classical way. The issue is studied in a model problem.

Voltage: Model Problem

- Coupled field-circuit simulation of circuit breaker requires network parameters: voltage, resistance, inductance

- Formulation
  → loss of control over $\underline{A}$ in Ω_M
  → loss of control over inductive voltage in potential

- Consider model problem: Cylinder with harmonic current excitation



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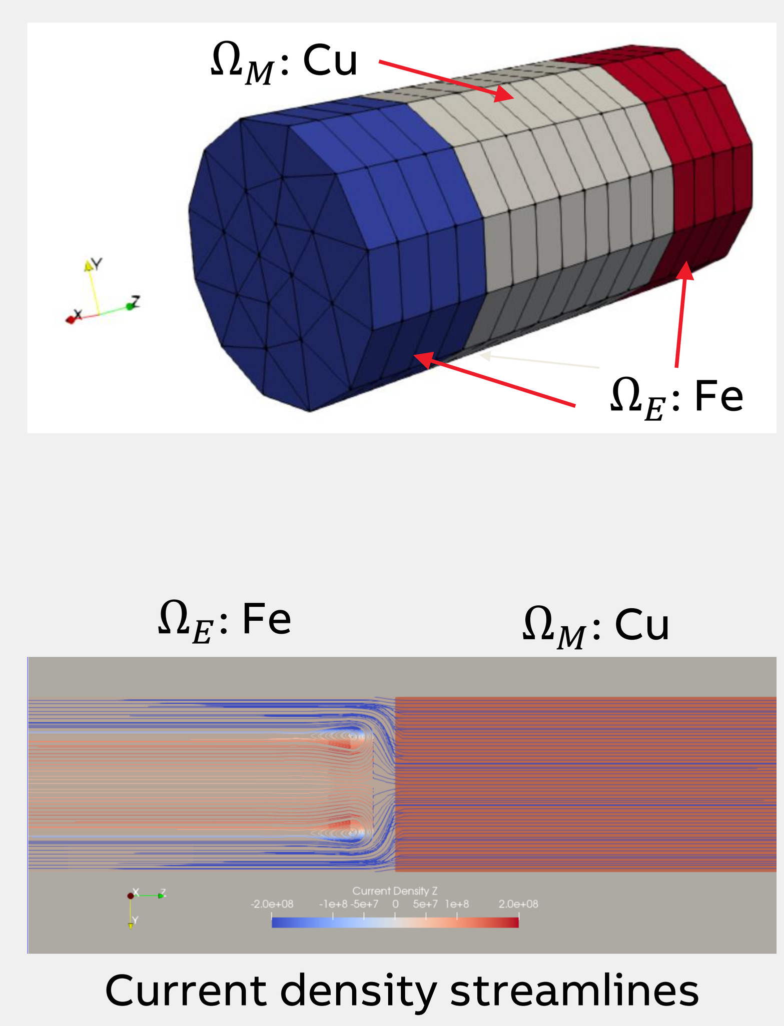
1-portion cylinder: analytic solution



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3-portion cylinder: formulation test

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Dedicated formulation requires non-standard post-processing of equivalent network parameters.

Equivalent network parameters

- Voltage U can be computed from power P, well defined
- Only 1 power → only 1 voltage

- Resistance R and inductance L relate to voltage U
- Only 1 equation (per time level) for 2 parameters R, L
  → use >1 time levels

- Power split Ohmic/magnetic is different from active/reactive (resistive/inductive)

Voltage from power

$$P = U\, I$$

$$P = P_{Ohm} + P_{mag}$$

$$P_{Ohm} = \int_\Omega \underline{j} \cdot \underline{E}\, \mathrm{d}V$$

$$P_{mag} = \int_\Omega \underline{H} \cdot \partial_t \underline{B}\, \mathrm{d}V = \mathrm{d}_t \left(\frac{1}{2} \int_\Omega \underline{H} \cdot \underline{B}\, \mathrm{d}V\right) = \mathrm{d}_t E_{mag}$$

Impedance fit

$$U(t_i) = R\, I(t_i) + L\, \mathrm{d}_t I(t_i)$$

$$\begin{pmatrix} I(t_{i-1}) & \mathrm{d}_t I(t_{i-1}) \\ I(t_i) & \mathrm{d}_t I(t_i) \end{pmatrix} \begin{pmatrix} R \\ L \end{pmatrix} = \begin{pmatrix} U(t_{i-1}) \\ U(t_i) \end{pmatrix}$$

more robust schemes possible

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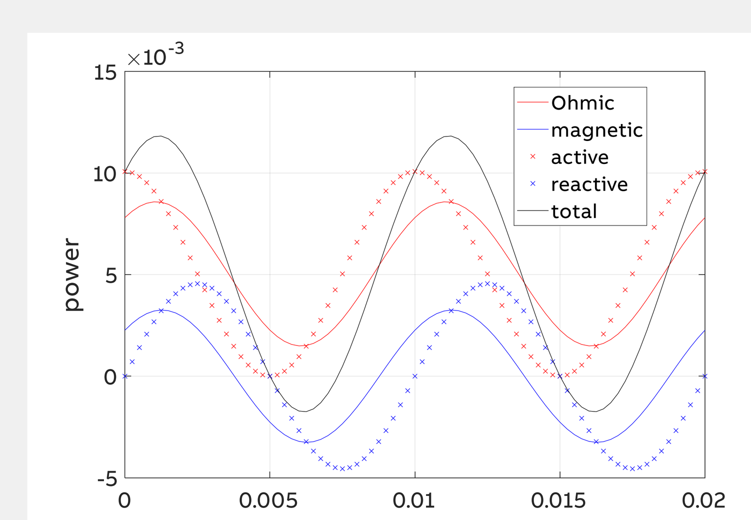
Voltage from power – impedance fit



$$P_{active} = R\, I^2 \neq P_{Ohm}$$

$$P_{reactive} = L\, I\, \mathrm{d}_t I \neq P_{mag}$$

At current zero, magnetic energy dissipates driving eddy currents.

→ Power balances inside domain, not observable at terminals.

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Iron cylinder power balance over time

arXiv pre-print