Magnetic Forces on Non-Linear Materials from FE-Solution

Christoph Winkelmann

ABB Corporate Research Switzerland, 5405 Baden-Dättwil

christoph.winkelmann@ch.abb.com

Abstract

In order to support its development of electric circuit breakers, ABB is developing tools for the simulation of electric arcs in these devices, coupling compressible flow, electromagnetism and kinematics. The electro-magnetism is treated as magneto-static in each time step, and solved with a tailored finite element solver co-developed with ETH Zürich [Cas2017], coupled to commercial tools for flow and kinematics. In order to account for the influence of magnetic forces on the kinematics of iron parts used e.g. for triggering of the interruption, we want to compute the magnetic force field from the finite element solution.
We suggest a procedure for computing this force field based on the weak divergence of an augmented Maxwell tensor proposed recently by Bossavit [Bos2015] for non-linear magnetic materials. Magnetic forces can be distinguished from (macroscopic) Lorentz forces, and permanent magnets can be included consistently. The distributional approach allows for recovery of first order convergence of total forces on bodies even for first order edge elements in a vector potential formulation. The computational cost is negligible compared to the FE-solution, because only local post-processing is required. Validation against commercial software shows competitive performance in convergence order, absolute accuracy and speed.

Context

ABB Circuit Breakers

- ABB develops electric circuit breakers: world leader, products from 2 kW to 2 GW
- ABB develops tools for the simulation of these
- interrupt current e.g. when short circuit occurs

Relevant Physics

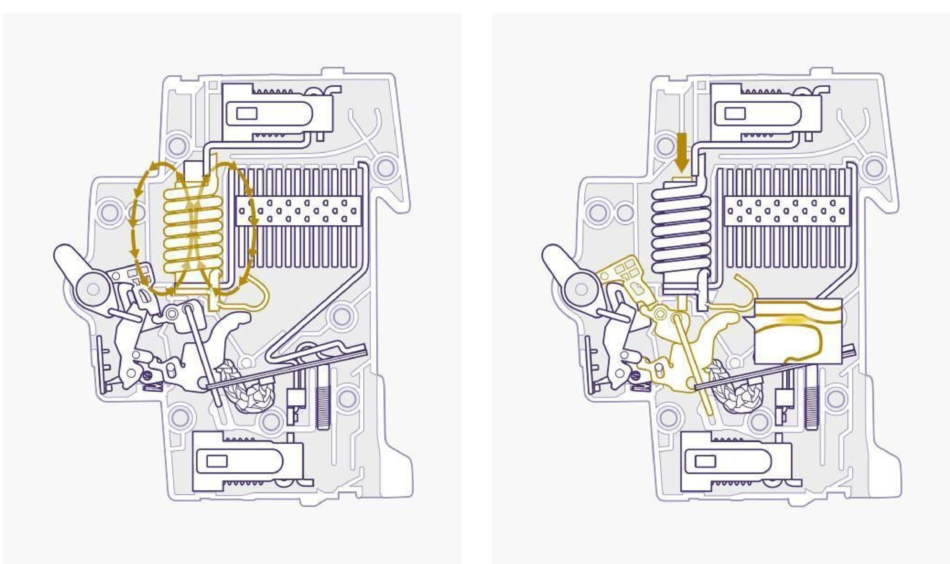

Figure 1: *Triggering of contact opening (left) and formation of arc (right) in a miniature circuit breaker.*

- short circuit ⇒ high current ⇒ magnetic force
- contacts open ⇒ plasma arc forms
- arc heated by ohmic losses and moved by Lorentz forces
- if arc extinguishes ⇒ current interrupted

Electromagnetics

Current conservation and magnetostatics on domain Ω:

$$\underline{\nabla}\cdot(-\sigma\underline{\nabla}\varphi) = 0 \qquad -\underline{\nabla}\varphi = \underline{E} \qquad \sigma\underline{E} = \underline{j}$$
$$\underline{\nabla}\times\left(\mu^{-1}\underline{\nabla}\times\underline{A}\right) = \underline{j} \qquad \underline{\nabla}\times\underline{A} = \underline{B} \qquad \mu^{-1}\underline{B} = \underline{H}$$

σ: electric conductivity	μ: magnetic permeability
φ: electric potential	$\underline{A}$: magnetic vector potential
$\underline{E}$: electric field	$\underline{B}$: magnetic flux density
$\underline{j}$: electric current density	$\underline{H}$: magnetic field strength

μ depends on $\|\underline{H}\|$, ⇒ non-linear material

Relevant quantities for the coupled arc simulation:

- ohmic heating power density $q = \underline{j}\cdot\underline{E}$
- Lorentz force density $\underline{f}_L = \underline{j}\times\underline{B}$
- magnetic force density $\underline{f}_M = ?$

Magnetic Forces

Microscopic Maxwell equations

Field, current, and force:

$$\begin{aligned}\underline{B} &= \mu_0\underline{H}\\ \underline{J} &= \mu_0^{-1}\underline{\nabla}\times\underline{B} = \underline{\nabla}\times\underline{H}\\ \underline{f} &= \underline{J}\times\underline{B} = \mu_0^{-1}(\underline{\nabla}\times\underline{B})\times\underline{B} = \ldots = \underline{\nabla}\cdot(\underline{\underline{T}}(\underline{B}))\end{aligned}$$

μ_0: magnetic permeability of vacuum
Total force $\underline{f}$ is Lorentz force due to total current density $\underline{J}$
Maxwell stress tensor: $\underline{\underline{T}}(\underline{B}) = \mu_0^{-1}\left(\underline{B}\otimes\underline{B} - 1/2\,\|\underline{B}\|^2\underline{\underline{\delta}}\right)$

Macroscopic Maxwell equations

Field, current, and force:

$$\begin{aligned}\underline{B} &= \mu\underline{H} = \mu_0(\underline{H}+\underline{M})\\ \underline{J} &= \mu_0^{-1}\underline{\nabla}\times\underline{B} = \underline{\nabla}\times\underline{H} + \underline{\nabla}\times\underline{M}\\ \underline{f} &= \underline{J}\times\underline{B} = \ldots = \underline{\nabla}\cdot\underline{\underline{T}}_A\end{aligned}$$

Augmented Maxwell stress tensor by Bossavit [Bos2015]:
$\underline{\underline{T}}_A = 1/2\,(\underline{B}\otimes\underline{H} + \underline{H}\otimes\underline{B}) - \phi\underline{\underline{\delta}}$
ϕ: magnetic co-energy, $\underline{\underline{\delta}}$: identity tensor

- Split field into $\underline{H}$ and magnetization $\underline{M}$
- Split total current $\underline{J}$ into macroscopic current $\underline{j} = \underline{\nabla}\times\underline{H}$ and microscopic current $\underline{\nabla}\times\underline{M}$
- Split total force $\underline{f}$ into (macroscopic) Lorentz force $\underline{f}_L = \underline{j}\times\underline{B}$ and magnetic force $\underline{f}_M = (\underline{\nabla}\times\underline{M})\times\underline{B}$.

Note: $\underline{f}_L \in L^2(\Omega)$ but $\underline{f}_M \notin L^2(\Omega)$ in general, because $\underline{M} \notin H(\mathrm{curl},\Omega)$ in general.

Force Density as a Distribution

$\underline{f} = \underline{\nabla}\cdot\underline{\underline{T}}_A$ in the sense of distributions:
$W : \underline{v} \to W(\underline{v}) = \langle\underline{f},\underline{v}\rangle_\Omega \quad \underline{v}\in H^1(\Omega)^3$

$$\Rightarrow W(\underline{v}) = -\int_\Omega \underline{\underline{T}}_A : \underline{\nabla}\,\underline{v}\ \mathrm{d}V + \int_{\partial\Omega}(\underline{\underline{T}}_A\cdot\underline{n})\cdot\underline{v}\ \mathrm{d}S$$

Interpretation as work W of force $\underline{f}$ under displacement $\underline{v}$
W_M = work of $\underline{f}_M$ ⇒ subtract contribution by Lorentz force:

$$W_M(\underline{v}) = W(\underline{v}) - \int_\Omega(\underline{j}\times\underline{B})\cdot\underline{v}\ \mathrm{d}V$$

Approximate Force Field from FE-Solution

- $\mathcal{T}$: (hybrid) mesh of domain Ω
- $\varphi_h \in V_h$: first order nodal FE-approximation of φ
- $\underline{A}_h \in R_h$: first order edge element FE-approximation of $\underline{A}$

Goal: Find element-wise constant approximate of magnetic force density $\hat{\underline{f}}_M$ such that $\int_\Omega \hat{\underline{f}}_M\cdot\underline{v}\ \mathrm{d}V \approx W_M(\underline{v}) \quad \forall\underline{v}\in V_h^3$

- choose $\underline{v}(\underline{x}) = \underline{e}w(\underline{x})$
- $\underline{e}\in I\!R^3$: spatially constant test vector
- w: scalar first order FE test function

$$\begin{aligned}W_M(\underline{e}w) = &-\int_\Omega(\underline{\underline{T}}_A\cdot\underline{\nabla}w)\cdot\underline{e}\ \mathrm{d}V + \int_{\partial\Omega}(\underline{\underline{T}}_A\cdot\underline{n})\cdot\underline{e}w\ \mathrm{d}S\\ &-\int_\Omega(\underline{j}\times\underline{B})\cdot\underline{e}w\ \mathrm{d}V \approx \int_\Omega(\hat{\underline{f}}_M w)\cdot\underline{e}\ \mathrm{d}V\end{aligned}$$

Compute $\underline{j}$ from φ_h, $\underline{B}$ and $\underline{\underline{T}}_A$ from $\underline{A}_h$
Formally drop $\underline{e}$ to obtain

$$\begin{aligned}\underline{F}(w) := &\int_\Omega -\underline{\underline{T}}_A\cdot\underline{\nabla}w - (\underline{j}\times\underline{B})w\ \mathrm{d}V + \int_{\partial\Omega}(\underline{\underline{T}}_A\cdot\underline{n})\,w\ \mathrm{d}S\\ \approx &\int_\Omega \hat{\underline{f}}_M w\ \mathrm{d}V\end{aligned}$$

Consider $w = w_i$ linked to node i with support ω_i

$$\underline{F}_i := \underline{F}(w_i) \approx \int_{\omega_i}\hat{\underline{f}}_M w_i\ \mathrm{d}V$$

- Interpret $\underline{F}_i$ as force on fuzzy body of extent ω_i around mesh node $\underline{x}_i$
- Distribute this node force conservatively and consistently to neighboring cells K
- Choose linear weighted distribution:
  $\hat{\underline{f}}_{M,K} := \hat{\underline{f}}_M\Big|_K = \sum_{j:K\subset\omega_j}\alpha_{Kj}\underline{F}_j \quad K\in\mathcal{T}$
- Conservation: $\alpha_{Ki} = |K|^{-1}\beta_{Ki}\left(\sum_{K'\subset\omega_i}\beta_{K'i}\right)^{-1}$
- Consistency: $\beta_{Ki} = \int_K \|\underline{M}\|^2 w_i\ \mathrm{d}V$

Convergence Behavior

Approach is implemented in our finite element solver **HyDi** (for **Hy**brid **Di**scontinuous finite elements) [Cas2017]

Benchmark

Geometry: Fig. 2

- current-carrying copper bar (10 mm cross section)
- non-linear iron yoke and bridge (5 mm cross section)

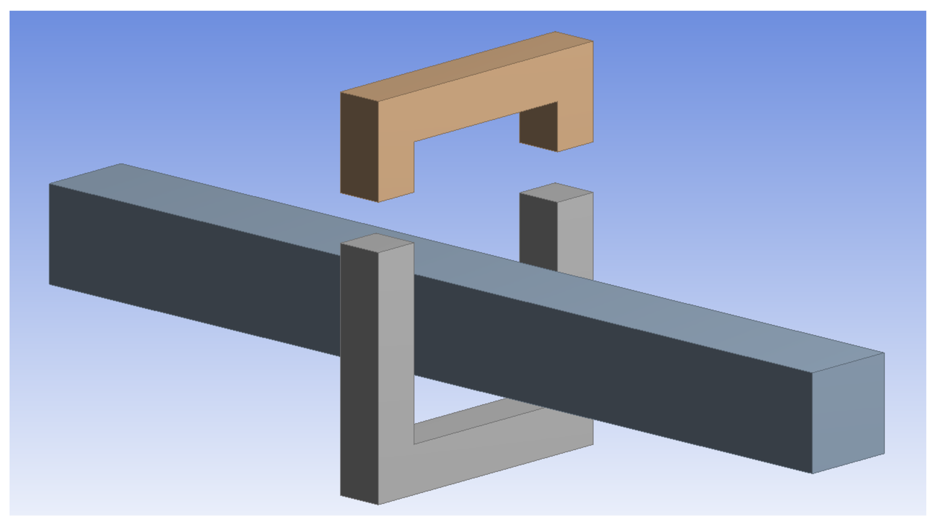

Figure 2: *Benchmark geometry*

Procedure:

- compute vertical force on iron bridge
- vary current, air gap, and software
- validate approach and study convergence

Results

General observations:

- magnetic forces in iron balance with Lorentz force in bar
- approach reproduces results of commercial software

Convergence observations (Fig. 3):

- first order convergence of total force on bridge
- efficiency can be increased by mesh grading
- more efficient than COMSOL despite lower order of elements

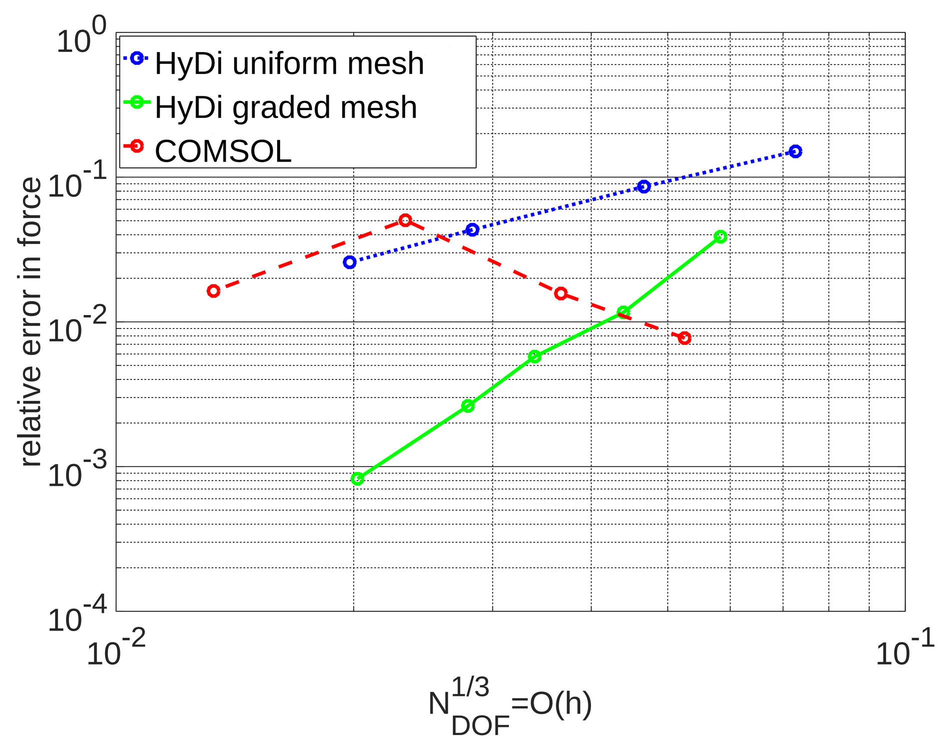

Figure 3: *Convergence of vertical force on iron bridge*

Overview of Further Results

Real Geometry

Validation versus ANSYS Maxwell for real geometry of low voltage breaker

Permanent Magnets

- Simple model: $\underline{B} = \mu_0(\underline{H}+\underline{M})$, $\underline{M}$ constant
- Verification by compass needle experiment

Limit of Magnetostatic Assumption

- 1D time dependent eddy-current simulation of $\underline{H}$ in yoke
- Periodic excitation, vary field and dimension
- Comparison of force to magnetostatic force
- Fit correction factor as average over one period
- Include fitted correction in solver

References

[Bos2015] Bossavit, Alain: Bulk Forces and Interface Forces in Assemblies of Magnetized Pieces of Matter. IEEE Transactions on Magnetics, Vol. 52, Issue 3, 2016 (extended preprint of 2015).
[Cas2017] Casagrande, Raffael: Discontinuous Finite Element Methods for Eddy Current Simulation. Dissertation, ETH Zürich, 2017, No. 24068.